The derivative in equation (6) is merely the slope of the boiling curve and $(R_T)^{-1}$ may be thought of as the effective heat-transfer coefficient between heat source and boiling surface. This relation is valid in all portions of the boiling curve. Since the slope of the boiling curve is positive in the nucleate and film regions, these regions are always stable operating areas. However, low thermal resistance is an absolute necessity for stable operation in the transition region.

Using only two inches of copper between heat source and surface reduces the effective coefficient to less than 1400 Btu/h ft^2 F (4415.5 J/m^{2}s) and makes it impossible to operate stably in the transition region with most fluids. Previous workers had at least that equivalent resistance in their equipment and often much more, especially where condensing steam was used. It is for this reason they were unable to obtain data over the entire transition region.

This investigation was carried out with equipment which satisfied the previously mentioned stability criterion. It contained 0.125 in (0.003175 m) of copper (gold plated) and utilized dropwise condensation of steam to minimize system thermal resistance.

CONCLUSIONS

It has been demonstrated that when the stability requirement

$$\frac{1}{R_T} \geqslant \frac{-\mathrm{d}Q_{\mathrm{out}}}{\mathrm{d}T_{\mathrm{sur}}}$$

is satisfied for the boiling equipment stable boiling can be obtained over the entire transition region.

Acknowledgements—The financial assistance of the Shell Oil Company, E. I. DuPont de Nemours & Company, and Atlantic Richfield Company is gratefully acknowledged by the authors.

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Int. J. Heat Mass Transfer. Vol. 19, pp. 698–700. Pergamon Press 1976. Printed in Great Britain

HEAT TRANSFER FROM A CONSTANT TEMPERATURE CIRCULAR CYLINDER IN CROSS-FLOW

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(*Received* 22 *January* 1973 *and in revised form* 4 *August* 1975)

NOMENCLATURE

- A, constant defined in equation (5);
- B, constant entering in equation (7);
- b, semiaxis of the ellipse defined in equation (1);
- C, constant entering in equation (6);
- Nu, Nusselt number;
- n, exponent used in equation (7);
- Pr, Prandtl number;
- Re, Reynolds number based on the diameter of the cylindrical wire;
- r, polar coordinate;
- T_1, normalized temperature in the outer region equal to $(T_b - T_\infty)/(T_a - T_\infty)$;
- T_2, normalized temperature in the inner region equal to $(T_a - T_b)/(T_a - T_\infty)$;
- T_a, temperature on the surface of the circular cylinder;
- T_b, temperature on the displaced boundary;
- T_∞, free stream temperature;
- $\mathbf{U}_1$, normalized velocity vector of the inviscid flow;
- ε, eccentricity of the ellipse defined in equation (1);
- θ, polar coordinate.

INTRODUCTION

THE PROBLEM of forced convective heat transfer from a circular cylinder to a transverse flow is a classical one. Beginning with Boussinesq's investigation in 1905 [1], many theoretical contributions to this problem followed. A review of the theoretical and experimental literature can be found in [2].

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The authors have undertaken measurements of turbulence in a Magneto-Fluid-Mechanic channel [2, 3]. It was found that neither the data taken in mercury by means of a hot film anemometer, nor Sajben's data [4] taken by using a hot wire anemometer could be interpreted by any of the existing theories or empirical laws. Such a discrepancy motivated the undertaking of a theoretical analysis.

THEORY

The present theoretical investigation assumes that the heat transfer to the viscous flow around a cylinder can be calculated by means of a hypothetical model which consists of the following two regions: A stationary fluid in the inner region and an inviscid flow outside. The outer region is denoted with the subscript 1 and the inner region with the subscript 2. The boundary between these two regions is chosen to be an ellipse. This is done to take into account the near wake.

$$r = \frac{b}{1+\varepsilon\cos\theta}. \tag{1}$$

By considering Oseen's solution on the limiting case of $Re \to 0$, the similarity solution on high Reynolds number, dimensional analysis arguments and some experimental data, the constants of equation (2) are determined as follows [2]:

$$\varepsilon = \frac{0.57\sqrt{Re}}{1+0.627Re} \tag{2}$$

$$b = [1+0.6469(Re\cdot Pr)^{-1/2}](1+\varepsilon). \tag{3}$$

The problem is then reduced to the solution of two simultaneous boundary value problems as follows: In the outer region we have

$$\mathbf{U}_1\cdot\nabla T_1 = \frac{2}{(Pr)(Re)}\nabla^2 T_1$$

$$T_1 \equiv \frac{T-T_\infty}{T_a-T_\infty} = 0 \quad \text{at} \quad r\to\infty$$

$$T_1 \equiv \frac{T_b-T_\infty}{T_a-T_\infty}, \quad \text{on} \quad r = \frac{b}{1+\varepsilon\cos\theta}$$

and in the inner region, where conduction is the prevailing mechanism:

$$\nabla^2 T_1 = 0 \quad \text{with} \quad T_2 = 0 \quad \text{on} \quad r = 1$$

$$T_2 = \frac{T_a-T_b}{T_a-T_\infty} \quad \text{on} \quad r = \frac{b}{1+\varepsilon\cos\theta} \quad \text{and} \quad T_b = T_b(\theta).$$

In the above U_1 is the normalized velocity in region 1, T_1, T_2 are normalized temperatures and a is the radius of the cylinder. By matching the temperature and the heat flux across the intermediate boundary, we determine the temperature both in the inner and the outer region as functions of r and θ. Consequently, the heat flux and the local Nusselt number on the surface of the cylinder are evaluated in forms of Fourier cosine series, and the mean Nusselt number is derived as

$$Nu = \frac{2A}{\sqrt{(2\pi)}+A\log b} \tag{4}$$

where

$$A = \{Pr\cdot Reb[1+(1-\varepsilon^2)^{1/2}](1+\varepsilon)\}^{1/2} \tag{5}$$

with b and ε defined by equations (4) and (3).

RESULTS

By using this formula, the heat transfer from a circular cylinder to air flow has been calculated from Reynolds number 10^{-2} to 9×10^5 and plotted against experimental data in Fig. 1. The agreement in the range of $0.1 < Re < 3\times10^4$ is excellent. The deviation of the theoretical values from the data can be attributed to the turbulence boundary layer on the surface which destroys the model derived from the concept of laminar boundary-layer displacement thickness in the high Reynolds number range, and the natural convection in the low Reynolds number range [2].

Dennis *et al.* [5] have investigated the forced convection from a circular cylinder numerically. Their results on the mean Nusselt number are very close to the results of the present theory. In fact in the range $40 < Re < 0.1$ the difference is less than 7%.

The local Nusselt numbers around a circular cylinder in cross-flow has also been verified with the existing data. The calculated values are plotted against the data by Eckert and Soehngen [6] in Fig. 2. The shapes of these curves are also found identical to what Dennis *et al.* obtained (Fig. 2 of the cited paper).

The theory was further extended to accommodate the influence of natural convection. The results generally agree with the data of Van der Hegge Zijnen [7]. Again, [2] can be consulted.

For the hot film probe of an anemometer in an electrically conducting fluid, equation (4) is modified to be

$$Nu = \frac{2A}{\sqrt{(2\pi)}+A(\log b+C)} \tag{6}$$

where C is a constant. This formula has been verified with the data taken in the channel described in [2] where mercury was used as the working fluid.

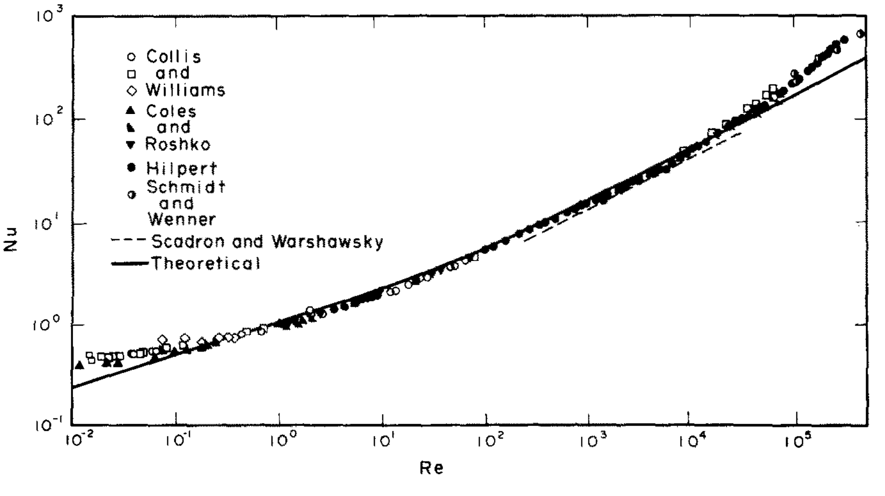

FIG. 1. Heat transfer from a circular cylinder to a cross-flow of air.

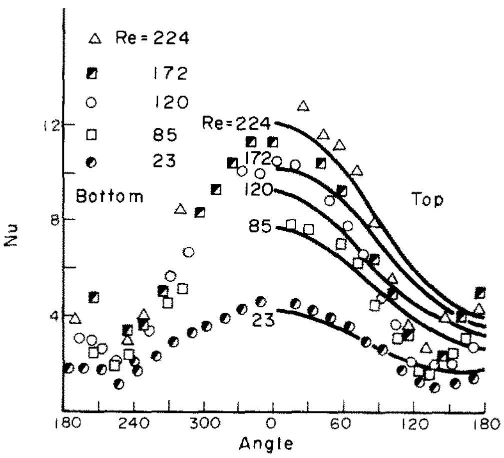

FIG. 2. Heat transfer around a circular cylinder to a cross-flow of air.

APPLICATION IN HOT FILM ANEMOMETRY

For measurements of turbulence, the local slope of the calibration curve of a hot wire anemometer is commonly used to compute such quantities as turbulence intensities, Reynolds stresses, etc. The calibration curve is fitted to the calibration data either graphically or with the help of a statistical criterion. The latter involves the finding of at least two out of the three constants, A, B, and n, of the widely used empirical formula

$$Nu = A + B \cdot Re^n. \quad (7)$$

Even so, it is doubtful that the fitted equation gives the correct first deviatives over the entire range of interest. Therefore, the graphical method is generally favored. However, the accuracy of the graphical method depends heavily on the individual's personal judgment and experience; moreover, the graphical method cannot be used to evaluate a large amount of data with the help of a computer. For computer calculations, a theoretical equation representing the essential behavior of the hot wire anemometer, with as few undetermined local constants as possible, is needed. It is believed that the present theory through equation (6) satisfies this need.

The deviation of the calibration curve of a hot wire anemometer from time to time is generally attributed to the contamination of impurities on the wire surface. It can be regarded as a coating of certain thickness. In equation (6), ε, b and A are given by equations (2), (3) and (5). The local constant C can be found by best fit of equation (6) to the calibration data.

This method has been actually used to process the data obtained in the mercury channel described in [2]. The results were comparatively better than those obtained with the statistical method.

Acknowledgement—The authors wish to acknowledge the financial assistance of the National Science Foundation under Grant GK-13552.

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Int. J. Heat Mass Transfer. Vol. 19, pp. 700–702. Pergamon Press 1976. Printed in Great Britain

EFFECT OF STALL LENGTH ON HEAT TRANSFER IN REATTACHED REGION BEHIND A DOUBLE STEP AT ENTRANCE TO AN ENLARGED FLAT DUCT

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(*Received* 4 *September* 1975)

NOMENCLATURE

h, step height;
L, entrance height;
$Nu_{d,\max}$, maximum value of local Nusselt number based on hydraulic diameter;
$Nu_{L,\max}$, maximum value of local Nusselt number, $\alpha_{\max}L/\lambda$;
Re_d, Reynolds number based on hydraulic diameter;
Re_L, Reynolds number, uL/ν;
q_w, wall heat flux;
u, flow velocity at entrance;
x_R, overall stall length.

Greek symbols

α, heat-transfer coefficient;
λ, thermal conductivity;
ν, kinematic viscosity.

INTRODUCTION

IN A FLOW region with an abruptly enlarged area change of a tube or of a duct, it is well known that the separated and the reattached regions occur. Especially, heat-transfer problems for such a flow geometry have been studied by several investigators. For a circular cross-sectional duct, Krall and Sparrow [1] and Ede *et al.* [2] have reported